Journal of Statistical Physics, Vol. 111, Nos. 1/2, April 2003 (© 2003)

Book Review: *Path Integrals in Quantum Mechanics, Statistics, Polymer Physics and Financial Markets*

Path Integrals in Quantum Mechanics, Statistics, Polymer Physics and Financial Markets. Prof. Dr. Hagen Kleinert, 3rd extended edition, World Scientific Publishing, Singapore

Richard Feynman, introduced path integral quantization in a further development of notions developed by Paul Dirac, in the dawn of quantum field theory in the second half of the twentieth century. Although this approach to quantization has had a tremendous impact in furthering our understanding of the fundamental interactions that govern the dynamics of the constituents of matter, it is still not often encountered as a "standard" part of a physics curriculum. This is a rather lamentable state of affairs as this is a method with a strong emphasis on concepts that closely entangle classical and quantum mechanics. It, in a sense, would represent an suitable didactical starting point for the discussion of quantum physics if it weren't for the mathematical complexity involved when contrasted with the standard approaches based upon the Schrödinger equation. One example where this becomes particularly evident is in the path integral quantization of the hydrogen atom. Rephrasing quantum mechanics in the language of transition amplitudes is tempting. Not only does it reveal a close proximity of quantum mechanics to quantum field theory, but also is in close correspondence with the classical mechanics of particles "moving from A to B."

Kleinert's book on Path Integrals in Quantum Mechanics, Statistics, and Polymer Physics that is now going into a 3rd revised edition, presents the reader with a very complete and very thorough discussion of path integration. A particularly important issue is this quantization of the hydrogen atom. Though being a crucial problem, and as such has obtained a standard place in any textbook on undergraduate Quantum Mechanics, in path integral terms its solution has for a long time seemed forbiddingly difficult. Kleinert devotes and entire chapter to these, and similar central potential systems. His continuous strife for clarity and completeness is all the more helpful here, as the path integral quantization of this pivotal

0022-4715/03/0400-0497/0

system is complicated. Although it hardly seems worth the effort when starting on this endeavour, the reward at the end is great. Questions that remain basically unanswered in the standard approaches simply because they cannot be asked in a meaningful way, are resolved in terms of the collapse of classical paths. But beyond that, the student will have learned a great deal about quantizing systems in general coordinates, very valuable for the application to systems in curved spaces or spaces with topological defects.

In the more advanced chapters many issues are dealt with that are challenging and rewarding for graduates students. Whether it is quantum mechanics in curved space, possibly with torsion, or whether one wishes to focus on condensed matter applications ranging from tunnelling to anyons and the day of metastable states; it is all there. Finally in the 3rd edition a new, extensive and, again, rather complete chapter has been added on the use of path integration techniques in the analysis of financial markets. This chapter would do well in any high-level course on stochastic financial models and is a wonderful occasion for candidate mathematical and theoretical physicists to realize what great potential there hides still in the methodologies and techniques that have been developed within their discipline.

The book offers such a wealth of topics; that it becomes straightforward for a lecturer to select material, both for advanced undergraduate as well as graduate courses in theoretical physics. An advanced undergraduate course, building upon a standard previous course in operator quantum mechanics, could start out with the first few chapters in path integration, discussing its derivation from ordinary quantum mechanics and the evaluation on Gaussian path integrals in real as well as imaginary time. In the 3rd edition this part has been significantly extended to cover a more complete treatment of functional determinants and scattering amplitudes. In the courses for which I use it, I always added at this point an excursion into the field theoretic context of path integral quantization. In the new third edition, Kleinert has taken care of this and thus fitting the field theory excursion neatly into the main body of the material. The following chapters treat the role of correlation functions, the Feynman diagram representation of the perturbative expansion of more involved potentials and variational approximation methods in great detail. A short undergraduate course would probably have then focus on the material concerning the semi-classical time evolution amplitudes and the path integral quantization of the hydrogen atom. The new section in the 3rd edition on Bose–Einstein condensation will however make every lecturer include this bit as well. Already the first half of the book contains a wealth of material.

This book is a major step towards bringing this approach to quantum physics onto the same educational footing as the Schrödinger equation that standard texts focus on. It profits from the clarity and conciseness that is also a hallmark of Kleinert's scientific papers. I have used this book in advanced undergraduate classes and received a very positive response from the students. But although they generally found its completeness and clarity helpful in their studies, the sheer magnitude may be a little bit daunting. All of these statements would have already been appropriate for the second edition, however the third addition comes with numerous extensions and a further deepening of path integration techniques. In a certain sense the book may even suffer from its completeness. A task that lies ahead of those interested in the didactics of path integration is, to use the abundant material in this volume and carefully extract the necessities for a true undergraduate course.

I would say this volume is highly recommendable for any student considering majoring in theoretical physics or chemistry, and an absolute must for any lecturer in this area. Researchers in physics education that are willing to take up the challenge of bringing path integration another step closer to becoming "standard" college material, will probably find this volume an excellent point of departure. In fact, I don't know of any excuse not to have your own copy.

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